

1

次の値を求めよ。

- (1) 10^0
- (2) 2^{-4}
- (3) $(-2)^{-3}$
- (4) $\left(-\frac{1}{3}\right)^{-2}$
- (5) 0.5^{-4}

2

次の計算をせよ。ただし、 $a \neq 0$, $b \neq 0$ とする。

- (1) $a^{-3}a^5$
- (2) $(a^{-2})^3$
- (3) $(a^{-1})^3 \times (a^{-2})^2$
- (4) $(a^3)^{-2} \div a^{-4}$
- (5) $a^2b \div a^{-1}b^3$
- (6) $(a^{-1}b)^{-2} \div (a^3b^{-1})^2$

3

次の計算をせよ。

- (1) $2^0 \div 2^{-4}$
- (2) $\sqrt[4]{64} \div \sqrt[4]{4}$
- (3) $0.125^{-\frac{2}{3}}$
- (4) $(8^{\frac{1}{2}} \times 4^{\frac{1}{4}})^{\frac{1}{2}}$
- (5) $\sqrt{6} \times \sqrt[4]{54} \div \sqrt[4]{6}$
- (6) $\left\{\left(\frac{5}{2}\right)^{-\frac{2}{3}}\right\}^{\frac{9}{2}} \div 5^{-3}$
- (7) $\sqrt{\sqrt[3]{2}} \times \sqrt[3]{\sqrt{32}}$

4

次の計算をせよ。

- (1) $\sqrt[3]{81} - \sqrt[3]{-3} + \sqrt[3]{-24}$
- (2) $\sqrt[3]{\sqrt{32}} \times \sqrt{8} \div \sqrt[3]{-16}$
- (3) $\sqrt[3]{54} + 5\sqrt[3]{-2} + \sqrt[3]{16}$
- (4) $\frac{8}{3}\sqrt[6]{9} + \sqrt[3]{-24} + \sqrt[3]{\frac{1}{9}}$

解説

- (1) $10^0 = 1$
- (2) $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$
- (3) $(-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$
- (4) $\left(-\frac{1}{3}\right)^{-2} = \frac{1}{\left(-\frac{1}{3}\right)^2} = \frac{1}{\frac{1}{9}} = 9$
- (5) $0.5^{-4} = \left(\frac{1}{2}\right)^{-4} = \frac{1}{\left(\frac{1}{2}\right)^4} = \frac{1}{\frac{1}{16}} = 16$

解説

- (1) $a^{-3}a^5 = a^{-3+5} = a^2$
- (2) $(a^{-2})^3 = a^{-2 \cdot 3} = a^{-6}$
- (3) $(a^{-1})^3 \times (a^{-2})^2 = a^{-3} \times a^{-4} = a^{-3+(-4)} = a^{-7}$
- (4) $(a^3)^{-2} \div a^{-4} = a^{-6} \div a^{-4} = a^{-6-(-4)} = a^{-2}$
- (5) $a^2b \div a^{-1}b^3 = a^{2-(-1)}b^{1-3} = a^3b^{-2}$
- (6) $(a^{-1}b)^{-2} \div (a^3b^{-1})^2 = (a^{-1})^{-2}b^{-2} \div (a^3)^2(b^{-1})^2 = a^2b^{-2} \div a^6b^{-2} = a^{2-6}b^{-2-(-2)} = a^{-4}b^0 = a^{-4}$

解説

- (1) $2^0 \div 2^{-4} = 2^{0-(-4)} = 2^4 = 16$
- (2) $\sqrt[4]{64} \div \sqrt[4]{4} = \sqrt[4]{\frac{64}{4}} = \sqrt[4]{16} = \sqrt[4]{2^4} = 2$
- (3) $0.125^{-\frac{2}{3}} = \frac{1}{0.125^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{0.125})^2} = \frac{1}{(\sqrt[3]{0.5^3})^2} = \frac{1}{0.5^2} = 4$

別解 $0.125^{-\frac{2}{3}} = (0.5^3)^{-\frac{2}{3}} = 0.5^{-2} = 4$

- (4) $(8^{\frac{1}{2}} \times 4^{\frac{1}{4}})^{\frac{1}{2}} = \{(2^3)^{\frac{1}{2}} \times (2^2)^{\frac{1}{4}}\}^{\frac{1}{2}} = (2^{\frac{3}{2} + \frac{1}{2}})^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2$
- (5) $\sqrt{6} \times \sqrt[4]{54} \div \sqrt[4]{6} = \sqrt{6} \times \sqrt[4]{\frac{54}{6}} = \sqrt{6} \times \sqrt[4]{3^2} = \sqrt{6} \times \sqrt{3} = 3\sqrt{2}$
- (6) $\left\{\left(\frac{5}{2}\right)^{-\frac{2}{3}}\right\}^{\frac{9}{2}} \div 5^{-3} = \left(\frac{5}{2}\right)^{-\frac{2}{3} \cdot \frac{9}{2}} \times 5^3 = \left(\frac{5}{2}\right)^{-3} \times 5^3 = \frac{2^3}{5^3} \times 5^3 = 8$
- (7) $\sqrt{\sqrt[3]{2}} \times \sqrt[3]{\sqrt{32}} = \sqrt[6]{2} \times \sqrt[6]{32} = \sqrt[6]{2 \times 32} = \sqrt[6]{2^6} = 2$

別解 $\sqrt{\sqrt[3]{2}} \times \sqrt[3]{\sqrt{32}} = (2^{\frac{1}{3}})^{\frac{1}{2}} \times \{(32)^{\frac{1}{2}}\}^{\frac{1}{3}} = 2^{\frac{1}{3} \cdot \frac{1}{2}} \times (2^5)^{\frac{1}{2} \cdot \frac{1}{3}} = 2^{\frac{1}{6}} \times 2^{\frac{5}{6}} = 2^{\frac{1}{6} + \frac{5}{6}} = 2$

解説

(1) $\sqrt[3]{81} - \sqrt[3]{-3} + \sqrt[3]{-24} = \sqrt[3]{3^3 \cdot 3} - (-\sqrt[3]{3}) - \sqrt[3]{2^3 \cdot 3}$
 $= 3\sqrt[3]{3} + \sqrt[3]{3} - 2\sqrt[3]{3} = (3+1-2)\sqrt[3]{3} = 2\sqrt[3]{3}$

参考 n が奇数のとき $\sqrt[n]{-a} = -\sqrt[n]{a}$

- (2) $\sqrt[3]{\sqrt{32}} \times \sqrt{8} \div \sqrt[3]{-16} = 32^{\frac{1}{6}} \times 8^{\frac{1}{2}} \div (-16^{\frac{1}{3}}) = (2^5)^{\frac{1}{6}} \times (2^3)^{\frac{1}{2}} \div \{-(2^4)^{\frac{1}{3}}\}$
 $= 2^{\frac{5}{6}} \times 2^{\frac{3}{2}} \times (-2^{-\frac{4}{3}}) = -2^{\frac{5}{6} + \frac{3}{2} - \frac{4}{3}} = -2^1 = -2$
- (3) $\sqrt[3]{54} + 5\sqrt[3]{-2} + \sqrt[3]{16} = \sqrt[3]{3^3 \cdot 2} + 5 \cdot (-\sqrt[3]{2}) + \sqrt[3]{2^3 \cdot 2}$
 $= 3\sqrt[3]{2} - 5\sqrt[3]{2} + 2\sqrt[3]{2} = (3-5+2)\sqrt[3]{2} = 0$
- (4) $\frac{8}{3}\sqrt[6]{9} + \sqrt[3]{-24} + \sqrt[3]{\frac{1}{9}} = \frac{8}{3}\sqrt[6]{3^2} + (-\sqrt[3]{2^3 \cdot 3}) + \sqrt[3]{\frac{3}{3^3}}$
 $= \frac{8}{3}(\sqrt[6]{3})^2 - \sqrt[3]{2^3 \cdot 3} + \frac{\sqrt[3]{3}}{\sqrt[3]{3^3}}$
 $= \frac{8}{3}\sqrt[3]{3} - 2\sqrt[3]{3} + \frac{1}{3}\sqrt[3]{3}$
 $= \left(\frac{8}{3} - 2 + \frac{1}{3}\right)\sqrt[3]{3} = \sqrt[3]{3}$